

Notes on soft breaking of BRST symmetry in the Batalin-Vilkovisky formalism

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Abstract

We have proved the nilpotency of the operators which describe the gauge dependence of the generating functionals of Green's functions for the gauge theories with the soft breaking of BRST symmetry in the Batalin-Vilkovisky formalism.

1 Introduction

In our paper we consider the problems related to the dependence of the Green's functions on the gauge in the soft breaking of BRST symmetry in the Batalin-Vilkovisky formalism [1] proposed in our previous papers [2], [3].

This breakdown in Yang-Mills theories is connected with a restriction of the domain of integration in the functional integral due to the Gribov horizon [4] and introducing of the Gribov-Zwanziger action [5],[6]. Note, the investigations for the theories above have been performed, as a rule, in the Landau gauge only (see, e.g., [7] and references therein).

At the same time, it is well known fact that the physical quantities and, in particular, S-matrix, can be calculated in different gauges in the framework of the Batalin-Vilkovisky of quantization of gauge theories, but they must not depend on a choice of the gauge condition. Recently, in Refs. [2], [3] a generalization of the definition of soft breaking of BRST symmetry valid for general gauge theories in arbitrary gauges within field-antifield formalism has been proposed.

The aim of the paper is the detailed elaboration of the properties of operators used in Ref. [3] for expressing the variations of generating functional of Green's functions under variation of the gauge condition.

We will use the condensed DeWitt's notations [8]. Derivatives with respect to sources and antifields are taken from the left, while those with respect to fields are taken from the right. The Grassmann parity of any quantity A in case of its homogeneity is denoted as $\varepsilon(A)$.

2 Odd Operators in BV quantization scheme with soft Breaking of BRST symmetry

Consider the configuration space parameterized by the fields $\Phi \equiv \{\Phi^A\} = \{A^i, \dots\}$ with $\varepsilon(\Phi^A) = \varepsilon_A$, where the dots indicate the full set of additional to A^i fields of this theory in the BV method in dependence on its reducibility stage. Then for each field Φ^A of this total configuration space, one should introduce the corresponding antifield Φ^* with opposite Grassmann parities to that of the corresponding field Φ^A $\Phi^* \equiv \{\Phi_A^*\} = \{A_i^*, \dots\}$ with $\varepsilon(\Phi_A^*) = \varepsilon_A + 1$.

In Ref. [3] it was shown that for the generating functional of Green's functions $Z(J, \Phi^*)$,

$$Z(J, \Phi^*) = \int D\Phi \exp \left\{ \frac{i}{\hbar} (S(\Phi, \Phi^*) + J_A \Phi^A) \right\}, \quad (1)$$

with action $S(\Phi, \Phi^*)$, which is additive extension of the non-degenerate gauge-fixing action $S_{ext}(\Phi, \Phi^*)$ by the bosonic functional $M(\Phi, \Phi^*)$, the variation of $Z(J, \Phi^*)$ induced by variation

of the gauge is written in the form

$$\begin{aligned} \delta Z(J, \Phi^*) &= \frac{i}{\hbar} \left[\left(J_A + M_A \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \right) \left(\frac{\delta}{\delta \Phi_A^*} - \frac{i}{\hbar} M^{A*} \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \right) \delta \Psi \left(\frac{\hbar}{i} \frac{\delta}{\delta J} \right) + \right. \\ &\quad \left. + \delta M \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \right] Z(J, \Phi^*). \end{aligned} \quad (2)$$

At deriving (2), we have taken into account that functionals $S_{ext}(\Phi, \Phi^*)$ and $(-M)$ satisfy the quantum master-equations of the BV method, J_A appears by the sources to the fields Φ^A , $\varepsilon(J_A) = \varepsilon_A$ and the notations

$$M_A \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \equiv \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^A} \Big|_{\Phi \rightarrow \frac{\hbar}{i} \frac{\delta}{\delta J}}, \quad M^{A*} \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \equiv \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi_A^*} \Big|_{\Phi \rightarrow \frac{\hbar}{i} \frac{\delta}{\delta J}}$$

were introduced. Here $M = M(\Phi, \Phi^*)$ plays the role of the functional, which describes the soft breaking of BRST symmetry in [3].

Let us introduce the odd operator \hat{q} :

$$\hat{q} = \left(J_A + M_A \right) \left(\frac{\delta}{\delta \Phi_A^*} - \frac{i}{\hbar} M^{A*} \right), \quad (3)$$

which contains non-vanishing terms $M_A \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right)$, $M^{A*} \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right)$ that differs it from the analogous operator considered in [2]. Then $\delta Z(J, \Phi^*)$ in (2) can be written in the form

$$\delta Z(J, \Phi^*) = \frac{i}{\hbar} \left[\hat{q} \delta \Psi \left(\frac{\hbar}{i} \frac{\delta}{\delta J} \right) + \delta M \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \right] Z(J, \Phi^*).$$

Let us prove the nilpotency of the operator \hat{q} , i.e. that, $\hat{q}^2 = 0$.

To do this, we will use for the shortness the following notations:

$$M_A = M_A \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right), \quad M^{A*} = M^{A*} \left(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right).$$

The square of \hat{q} may be directly presented as a sum of four operators

$$\begin{aligned} \hat{q}^2 &= \left[\left(J_A + M_A \right) \left(\frac{\delta}{\delta \Phi_A^*} - \frac{i}{\hbar} M^{A*} \right) \right]^2 \equiv \sum_{i=1}^4 D_i = \\ &= \left(J_A + M_A \right) \frac{\delta}{\delta \Phi_A^*} \left(J_B + M_B \right) \frac{\delta}{\delta \Phi_B^*} - \frac{i}{\hbar} \left(J_A + M_A \right) \frac{\delta}{\delta \Phi_A^*} \left(J_B + M_B \right) M^{B*} - \\ &- \frac{i}{\hbar} \left(J_A + M_A \right) M^{A*} \left(J_B + M_B \right) \frac{\delta}{\delta \Phi_B^*} + \left(\frac{i}{\hbar} \right)^2 \left(J_A + M_A \right) M^{A*} \left(J_B + M_B \right) M^{B*}. \end{aligned} \quad (4)$$

Consider the first term in the decomposition (4),

$$D_1 = \left(J_A + M_A \right) \frac{\delta M_B}{\delta \Phi_A^*} \frac{\delta}{\Phi_B^*} + \left(J_A + M_A \right) \left(J_B + M_B \right) \frac{\delta}{\delta \Phi_B^*} \frac{\delta}{\delta \Phi_A^*} (-1)^{\varepsilon_A + 1}. \quad (5)$$

In turn, the second summand in D_1 (5) has the form

$$(-1)^{\varepsilon_A + 1} \left(J_A J_B + M_A M_B + J_A M_B + (-1)^{\varepsilon_A \varepsilon_B} J_B M_A + \frac{\hbar}{i} M_{AB} \right) \frac{\delta}{\delta \Phi_B^*} \frac{\delta}{\delta \Phi_A^*}, \quad (6)$$

where we have taken into account that $[M_A, J_B] = \frac{\hbar}{i} M_{AB}$, determined as,

$$M_{AB} = \frac{\delta^2 M(\Phi, \Phi^*)}{\delta \Phi^A \delta \Phi^B} \Big|_{\Phi \rightarrow \frac{\hbar}{i} \frac{\delta}{\delta J}} \quad \text{and} \quad M_{AB} = (-1)^{\varepsilon_A \varepsilon_B} M_{BA} . \quad (7)$$

Note, the generalized symmetry properties of the expression in the brackets in (6) and the second derivative with respect to antifields are opposite under replacement of indices $A \leftrightarrow B$. From this fact it follows that (6) is equal to zero and D_1 term is determined only by the first term. Turning to the summand D_2 we see that after rearranging of the derivatives with respect to antifields D_2 takes the form

$$\begin{aligned} D_2 = & -\frac{i}{\hbar} (J_A + M_A) \frac{\delta M_B}{\delta \Phi_A^*} M^{B*} - \frac{i}{\hbar} (J_A + M_A) (J_B + M_B) \frac{\delta M^{B*}}{\delta \Phi_A^*} (-1)^{(\varepsilon_A+1)\varepsilon_B} - \\ & - \frac{i}{\hbar} (J_A + M_A) (J_B + M_B) M^{B*} \frac{\delta}{\delta \Phi_A^*} (-1)^{\varepsilon_A+1}. \end{aligned} \quad (8)$$

Then, we can see taking into account of the generalized symmetry property

$$\frac{\delta M^{B*}}{\delta \Phi_A^*} = \frac{\delta M^{A*}}{\delta \Phi_B^*} (-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}, \quad (9)$$

that the second term in D_2 vanishes.

Next, for the term D_3 we have after sequence of the transformations

$$\begin{aligned} D_3 = & -\frac{i}{\hbar} (J_A + M_A) M^{A*} J_B \frac{\delta}{\delta \Phi_B^*} - \frac{i}{\hbar} (J_A + M_A) M^{A*} M_B \frac{\delta}{\delta \Phi_B^*} = \\ = & -\frac{i}{\hbar} (J_A + M_A) \left(\frac{\hbar}{i} M_{AB}^{A*} + J_B M^{A*} (-1)^{\varepsilon_B(\varepsilon_A+1)} \right) \frac{\delta}{\delta \Phi_B^*} - \\ & - \frac{i}{\hbar} (J_A + M_A) M^{A*} M_B \frac{\delta}{\delta \Phi_B^*} = \\ = & - (J_A + M_A) M_{AB}^{A*} \frac{\delta}{\delta \Phi_B^*} - \frac{i}{\hbar} (J_A + M_A) (J_B + M_B) M^{A*} \frac{\delta}{\delta \Phi_B^*} (-1)^{\varepsilon_B(\varepsilon_A+1)}, \end{aligned} \quad (10)$$

where we have used the notation

$$M_{AB}^{A*} = \frac{\delta^2 M(\Phi, \Phi^*)}{\delta \Phi_A^* \delta \Phi^B} \Big|_{\Phi \rightarrow \frac{\hbar}{i} \frac{\delta}{\delta J}}. \quad (11)$$

At last, transforming the fourth summand D_4 in (4) as follows,

$$\begin{aligned} D_4 = & \left(\frac{i}{\hbar} \right)^2 (J_A + M_A) \left(\frac{\hbar}{i} M_{AB}^{A*} + J_B M^{A*} (-1)^{\varepsilon_B(\varepsilon_A+1)} \right) M^{B*} + \\ & + \left(\frac{i}{\hbar} \right)^2 (J_A + M_A) M^{A*} M^{B*} M_B = \\ = & \frac{i}{\hbar} (J_A + M_A) M_{AB}^{A*} M^{B*} + \left(\frac{i}{\hbar} \right)^2 J_B J_A M^{A*} M^{B*} (-1)^{\varepsilon_B} + \\ & + \left(\frac{i}{\hbar} \right)^2 \left(\frac{\hbar}{i} M_{AB} + J_B M_A (-1)^{\varepsilon_A \varepsilon_B} \right) M^{A*} M^{B*} (-1)^{\varepsilon_B(\varepsilon_A+1)} + \\ & + \left(\frac{i}{\hbar} \right)^2 (J_A + M_A) M^{A*} M^{B*} M_B, \end{aligned}$$

we have finally,

$$\begin{aligned}
D_4 = & \frac{i}{\hbar} (J_A + M_A) M_B^{A*} M^{B*} + \left(\frac{i}{\hbar}\right)^2 J_B J_A M^{A*} M^{B*} (-1)^{\varepsilon_B} + \\
& + \frac{i}{\hbar} M_{AB} M^{A*} M^{B*} (-1)^{\varepsilon_B(\varepsilon_A+1)} + \left(\frac{i}{\hbar}\right)^2 J_B M_A M^{A*} M^{B*} (-1)^{\varepsilon_B} - \\
& - \left(\frac{i}{\hbar}\right)^2 J_A M_B M^{B*} M^{A*} (-1)^{\varepsilon_A} + \left(\frac{i}{\hbar}\right)^2 M_A M_B M^{A*} M^{B*} (-1)^{\varepsilon_B(\varepsilon_A+1)}.
\end{aligned} \tag{12}$$

It is easy to see that the second, third and sixth terms in the last expression identically vanish due to the generalized symmetry properties under changing of the indices $A \leftrightarrow B$. Indeed, the quantities $(J_B J_A)$, M_{AB} are generalized-symmetric ones, whereas $(M^{A*} M^{B*} (-1)^{\varepsilon_B})$ is generalized-antisymmetric, and $(M_A M^{A*})^2 \equiv 0$. Next, the sum of the fourth and fifth terms is equal to zero. Therefore D_4 is reduced to the first term in (12).

In view of the derivations above, we have the following final representations for D_1 , D_2 , D_3 , D_4 ,

$$D_1 = (J_A + M_A) \frac{\delta M_B}{\delta \Phi_A^*} \frac{\delta}{\delta \Phi_B^*}, \tag{13}$$

$$D_2 = -\frac{i}{\hbar} (J_A + M_A) \frac{\delta M_B}{\delta \Phi_A^*} M^{B*} - \frac{i}{\hbar} (J_A + M_A) (J_B + M_B) M^{B*} \frac{\delta}{\delta \Phi_A^*} (-1)^{\varepsilon_A+1}, \tag{14}$$

$$D_3 = -(J_A + M_A) M_B^{A*} \frac{\delta}{\delta \Phi_B^*} - \frac{i}{\hbar} (J_A + M_A) (J_B + M_B) M^{A*} \frac{\delta}{\delta \Phi_B^*} (-1)^{\varepsilon_B(\varepsilon_A+1)}, \tag{15}$$

$$D_4 = \frac{i}{\hbar} (J_A + M_A) M_B^{A*} M^{B*}. \tag{16}$$

From the Eqs. (13)-(16) we immediately obtain, first, the sum of the operator D_4 and the first term in D_2 is equal to zero, second, the sum of the operator D_1 and the first term in D_3 vanishes, third, the sum of the second terms in both operators D_2 and D_3 is equal to zero.

Thus, our statement, that $\hat{q}^2 = 0$ is completely proved.

As the consequence, we have simultaneously proved the nilpotency of the operator \hat{Q} , which is unitarily related to \hat{q}

$$\hat{Q} = \exp -\frac{i}{\hbar} W \hat{q} \exp \frac{i}{\hbar} W = \left(J_A + M_A \left(\frac{\delta W}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \right) \frac{\delta}{\delta \Phi_A^*}. \tag{17}$$

Remind [3], the operator \hat{Q} the dependence of the generating functional of connected Green's functions $W(J, \Phi^*) = \frac{\hbar}{i} \ln Z(J, \Phi^*)$ on the variation of the gauge in the representation

$$\delta W(J, \Phi^*) = \hat{Q} \delta \Psi \left(\frac{\delta W}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) + \delta M \left(\frac{\delta W}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right). \tag{18}$$

At last, for the generating functional of vertex Green's functions (effective action), which is obtained from $W(J, \Phi^*)$ by means of the Legendre transformation with respect to sources

J_A , $(\Gamma(\Phi, \Phi^*) = W(J, \Phi^*) - J_A \Phi^A)$ with average fields $\Phi^A = \frac{\delta W}{\delta J_A}$, the local (for $M = 0$) representation for the odd and nilpotent operator \hat{s}_q being equal to \hat{Q} , but acting on the space of average fields and antifields will be valid as well,

$$\delta\Gamma(\Phi, \Phi^*) = \hat{s}_q < \delta\Psi > + < \delta M > . \quad (19)$$

The angle brackets in the expressions (19) denote the averaging of the quantities and operators with respect to the functional $\Gamma(\Phi, \Phi^*)$, considered in details in [2], [3], whereas the operator \hat{s}_q itself has the form, presented in fact in [3], with using the left derivative with respect to Φ^A ,

$$\begin{aligned} \hat{s}_q = & -\left(\frac{\delta\Gamma}{\delta\Phi^A} - \hat{M}_A\right)\frac{\delta}{\delta\Phi_A^*} - (-1)^{\varepsilon_A}\left(\frac{\delta\Gamma}{\delta\Phi_A^*} - \hat{M}^{A*}\right)\frac{\delta_l}{\delta\Phi^A} \\ & - \frac{i}{\hbar}\left[\widehat{M}_A\frac{\delta\Gamma}{\delta\Phi_A^*} + \frac{\delta\Gamma}{\delta\Phi^A}\widehat{M}^{A*} - \widehat{M}_A\widehat{M}^{A*}, \Phi^B\right]\frac{\delta_l}{\delta\Phi^B} . \end{aligned} \quad (20)$$

where the sign $[,]$ means for supercommutator.

The nilpotency of the operators \hat{q} , \hat{Q} , \hat{s}_q proved here repeats the properties of their analogs in the Ref. [2], but in case of more general regularization than dimensional one and takes fundamental character reflecting the presence of the BRST symmetry in the theory but with its soft breaking.

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